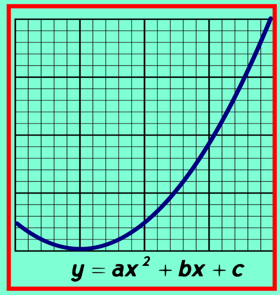
Math 125

Spring 2021

Lecture 25



Solve by Square-Root method:

$$(2x+5)^2 + 4 = -20$$

$$(2x+5)^2 = -24$$

$$2x+5 = \pm\sqrt{-24}$$

$$2x = -5 \pm \sqrt{4}\sqrt{6}\sqrt{-1}$$

$$2x = -5 \pm 2\sqrt{6}\, i$$

$$x = \frac{-5}{2} \pm \frac{\cancel{2}\sqrt{6}}{\cancel{2}}\, i$$

$$\boxed{x = \frac{-5}{2} \pm \sqrt{6}\, i}$$

$$\left\{ \frac{-5}{2} \pm \sqrt{6}\, i \right\}$$

Solve by Completing the Square method:

$x^2 + 5x + 4 = 0$

$x^2 + 5x = -4$

Lead. Coef. $= 1$

$\frac{1}{2} \cdot 5 = \frac{5}{2}$

$x^2 + 5x + \left(\frac{5}{2}\right)^2 = -4 + \left(\frac{5}{2}\right)^2$

$-4 + \frac{25}{4} = \frac{-4 \cdot 4}{4} + \frac{25}{4} = \frac{-16}{4} + \frac{25}{4}$

$x^2 + 5x + \frac{25}{4} = \frac{9}{4}$

$\left(x + \frac{5}{2}\right)^2 = \frac{9}{4}$

Use S.R.M.

$x + \frac{5}{2} = \pm\sqrt{\frac{9}{4}}$

$x + \frac{5}{2} = \pm \frac{3}{2}$

$x = \frac{-5}{2} \pm \frac{3}{2}$

$x = \frac{-5}{2} + \frac{3}{2} = \frac{-2}{2}$

$\boxed{x = -1}$

$x = \frac{-5}{2} - \frac{3}{2} = \frac{-8}{2}$

$\boxed{x = -4}$

$\boxed{\{-4, -1\}}$

Solve by Completing the Square method

$x^2 - 4x + 13 = 0$

$x^2 - 4x + (-2)^2 = -13 + (-2)^2$

$\frac{1}{2} \cdot (-4) = -2$

$x^2 - 4x + 4 = -9$

$(x-2)^2 = -9$

Use S.R.M.

$x - 2 = \pm\sqrt{-9}$

$\boxed{x = 2 \pm 3i}$

$\{2 \pm 3i\}$

Given $3x^2 - 5x - 8 = 0$

1) Find a, b, and c.

$a = 3$ $b = -5$ $c = -8$

2) Evaluate $b^2 - 4ac$

$b^2 - 4ac = (-5)^2 - 4(3)(-8) = 25 + 96 = \boxed{121}$

3) Solve by using the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{121}}{2(3)} = \frac{5 \pm 11}{6}$$

$x = \frac{5+11}{6} = \frac{16}{6}$ $\boxed{x = \frac{8}{3}}$

$x = \frac{5-11}{6} = \frac{-6}{6}$ $\boxed{x = -1}$

$$\boxed{\left\{-1, \frac{8}{3}\right\}}$$

Given $(3x + 4)(2x - 1) = 10$

1) Foil, Simplify, and write in $ax^2 + bx + c = 0$ Form

$6x^2 - 3x + 8x - 4 - 10 = 0$

$\boxed{6x^2 + 5x - 14 = 0}$

2) Find a, b, and c.

$a = 6$

$b = 5$

$c = -14$

3) Evaluate $b^2 - 4ac$

$b^2 - 4ac = 5^2 - 4(6)(-14)$

$= 361$

4) Use the quadratic formula to solve.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{361}}{2(6)} = \frac{-5 \pm 19}{12}$$

$x = \frac{-5+19}{12} = \frac{14}{12}$ $\boxed{x = \frac{7}{6}}$

$x = \frac{-5-19}{12} = \frac{-24}{12}$ $\boxed{x = -2}$

$$\left\{-2, \frac{7}{6}\right\}$$

The product of two consecutive odd integers is 35.

x & $x+2$

Find all such integers.

$x(x+2)=35$

$x^2+2x-35=0$

$x^2+2x+1^2=35+1^2$

$\frac{1}{2}\cdot 2=1$

$x^2+2x+1=36$

$(x+1)^2=36$

$x+1=\pm\sqrt{36}$

$x=-1\pm 6$

$x=-1+6$
$=5$

$x=-1-6$
$=-7$

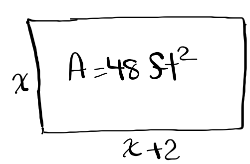
x	$x+2$
5	$5+2=7$
-7	$-7+2=-5$

5 & 7 OR −7 & −5

Length and width of a rectangular garden with area 48 ft² are two consecutive even integers.

x & $x+2$

Find dimensions of the garden.



Area = L · W

$x(x+2)=48$

$x^2+2x-48=0$

$a=1 \quad b=2 \quad c=-48$

$b^2-4ac=2^2-4(1)(-48)$
$=196$

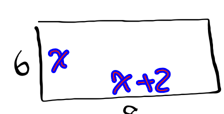
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2\pm\sqrt{196}}{2(1)}$$

$$=\frac{-2\pm 14}{2}$$

$$x=\frac{-2+14}{2}=\frac{12}{2}=6$$

$$x=\frac{-2-14}{2}=\frac{-16}{2}=\cancel{-8}$$



6 ft by 8 ft

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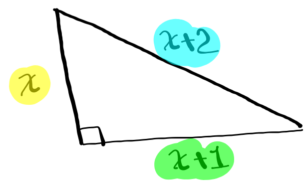
Find x:



Right - Triangle

$a^2 + b^2 = c^2$

Pythagorean Formula

$x^2 + \boxed{(x+1)^2} = \boxed{(x+2)^2}$

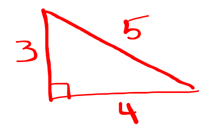
$\cancel{x^2} + x^2 + 2x + 1 = \cancel{x^2} + 4x + 4$

$\boxed{SG\ 16}$ ✓

$x^2 + 2x + 1 - 4x - 4 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$



$x - 3 = 0$

$x + 1 = 0$

$\boxed{x = 3}$ $\{3\}$ $\cancel{x = -1}$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

$25 = 25$ ✓

$ax^2 + bx + c = 0 \; ; \; a = 0$

Quadratic Equation

$b^2 - 4ac \Rightarrow$ Discriminant

$$b^2 - 4ac \begin{cases} > 0 \Rightarrow \text{Two Real Solutions} \\ = 0 \Rightarrow \text{One repeated Real Soln} \\ < 0 \Rightarrow \text{Two imaginary Soln.} \end{cases}$$

Given $3x^2 - 5x + 12 = 0$

1) a, b, c $a=3$ $b=-5$ $c=12$

2) Find $b^2 - 4ac = (-5)^2 - 4(3)(12) = 25 - 144$
$= \boxed{-119}$

3) Discuss the type of Solutions.
$b^2 - 4ac < 0 \Rightarrow$ Two imaginary Solutions

Given $4x^2 + 12x + 9 = 0$

1) a, b, c $a=4$ $b=12$ $c=9$

2) Find $b^2 - 4ac = 12^2 - 4(4)(9) = \boxed{0}$

3) Discuss the type of Solutions
$b^2 - 4ac = 0 \Rightarrow$ One repeated real Soln.

Given $(x+4)(x-2)=7$

1) write in $ax^2+bx+c=0$ form.

$$x^2-2x+4x-8-7=0$$

$$x^2+2x-15=0$$

2) find a, b, and c.

$a=1$, $b=2$, $c=-15$

3) Evaluate b^2-4ac

$$b^2-4ac=2^2-4(1)(-15)$$

$$=\boxed{64}$$

4) Discuss the type of Solutions.

$b^2-4ac>0 \Rightarrow$ Two Real Solutions

Doing Reverse:

Given Solutions

Find equation

Find a quadratic equation with Solutions -2 & 5 in $ax^2+bx+c=0$ form.

$x=-2$ $\qquad$ $x=5$

$x+2=0$ $\qquad$ $x-5=0$

$$(x+2)(x-5)=0$$

$$x^2-5x+2x-10=0$$

$$\boxed{x^2-3x-10=0}$$

Find a quadratic eqn in $ax^2+bx+c=0$ form with solutions $\frac{-1}{2}$ and $\frac{2}{3}$.

$x = \frac{-1}{2}$ $\qquad$ $x = \frac{2}{3}$

$2x = -1$ $\qquad$ $3x = 2$

$2x + 1 = 0$ $\qquad$ $3x - 2 = 0$

$$(2x+1)(3x-2) = 0$$

$$6x^2 - 4x + 3x - 2 = 0$$

$$\boxed{6x^2 - x - 2 = 0}$$

Find a quadratic equation in $ax^2+bx+c=0$ with solutions $2 \pm \sqrt{5}$.

$x = 2 + \sqrt{5}$ $\qquad$ $x = 2 - \sqrt{5}$

$x - 2 - \sqrt{5} = 0$ $\qquad$ $x - 2 + \sqrt{5} = 0$

$$\underbrace{(x - 2 - \sqrt{5})(x - 2 + \sqrt{5})}_{\text{Conjugates}} = 0$$

$$\underbrace{(x-2)^2}_{x^2 - 4x + 4} - (\sqrt{5})^2 = 0$$

$$x^2 - 4x + 4 - 5 = 0$$

$$\boxed{x^2 - 4x - 1 = 0}$$

Find a quadratic equation in $ax^2+bx+c=0$ form with Solutions $-3 \pm 4i$.

$x = -3 + 4i$ $\qquad$ $x = -3 - 4i$

$x + 3 - 4i = 0$ $\qquad$ $x + 3 + 4i = 0$

$$\underbrace{(x+3-4i)(x+3+4i)}_{\text{conjugates}} = 0$$

$$(\underbrace{x+3}_{x^2+6x+9})^2 - (4i)^2 = 0$$

$x^2 + 6x + 9 \quad - 16i^2 = 0$

$x^2 + 6x + 9 - 16(-1) = 0$

$x^2 + 6x + 9 + 16 = 0 \rightarrow \boxed{x^2+6x+25=0}$

Find a quadratic equation in $ax^2+bx+c=0$ form with Solutions $\frac{1}{3} \pm \frac{2}{3}i$.

$x = \frac{1}{3} + \frac{2}{3}i$ $\qquad$ $x = \frac{1}{3} - \frac{2}{3}i$

$3x = 1 + 2i$ $\qquad$ $3x = 1 - 2i$

$3x - 1 - 2i = 0$ $\qquad$ $3x - 1 + 2i = 0$

$$\underbrace{(3x-1-2i)(3x-1+2i)}_{\text{Conjugates}} = 0$$

$(3x-1)^2 - (2i)^2 = 0$

$9x^2 - 6x + 1 \quad - 4i^2 = 0$

$9x^2 - 6x + 1 - 4(-1) = 0$

$9x^2 - 6x + 1 + 4 = 0$

$\boxed{9x^2 - 6x + 5 = 0}$

Page 1 & 2 of SG 17

Solving quadratic equation by making Subs.

$$(x^2-8)^2 - 2(x^2-8) - 3 = 0$$

Notice x^2-8 is being used twice.

Let $u = x^2-8$

Our problem now becomes $u^2 - 2u - 3 = 0$

$(u-3)(u+1) = 0$

$u = 3 \quad u = -1$

when $u = 3$

$x^2 - 8 = 3$

$x^2 = 11$

$\boxed{x = \pm\sqrt{11}}$

when $u = -1$

$x^2 - 8 = -1$

$x^2 = 7$

$\boxed{x = \pm\sqrt{7}}$

$\{\pm\sqrt{11}, \pm\sqrt{7}\}$

$$(x^2+5)^2 + 4(x^2+5) - 5 = 0$$

Let $u = x^2+5$

Equation becomes $u^2 + 4u - 5 = 0$

$(u+5)(u-1) = 0$

$u = -5 \quad u = 1$

when $u = -5$

$x^2 + 5 = -5$

$x^2 = -10$

$x = \pm\sqrt{-10}$

$\boxed{x = \pm i\sqrt{10}}$

when $u = 1$

$x^2 + 5 = 1$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$\boxed{x = \pm 2i}$

$\{\pm 2i, \pm i\sqrt{10}\}$

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Solve $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 20 = 0$

$x^{\frac{2}{3}} = \left[x^{\frac{1}{3}}\right]^2$

$\left[x^{\frac{1}{3}}\right]^2 - \left[x^{\frac{1}{3}}\right] - 20 = 0$

Let $u = x^{\frac{1}{3}}$

$u^2 - u - 20 = 0$

$(u-5)(u+4) = 0$

$u = 5 \quad u = -4$

when $u = 5$

$x^{\frac{1}{3}} = 5$

$\sqrt[3]{x} = 5$

$\left(\sqrt[3]{x}\right)^3 = 5^3$

$\boxed{x = 125}$

when $u = -4$

$x^{\frac{1}{3}} = -4$

$\sqrt[3]{x} = -4$

$\left(\sqrt[3]{x}\right)^3 = (-4)^3$

$\boxed{x = -64}$

$\{-64, 125\}$

SG 17, work on Page 3 & 4.

Quadratic Function

$y = f(x)$

$f(x) = a(x-h)^2 + k$

$a \neq 0$

1) $a > 0$, $a < 0$

{Parabola}

2) Vertex (h, k)

3) Axis of Symmetry

$x = h$

3) Y-Int: Let $x = 0$, find y

4) x-Int: Let $y = 0$, find x

5) Graph.

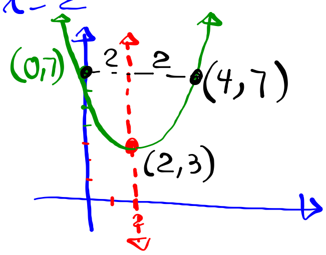
$f(x) = (x-2)^2 + 3$

$f(x) = a(x-h)^2 + k$

$a = 1$, opens upward

$h = 2$, $k = 3$ Vertex $(2,3)$

A.O.S. $x = 2$



Y-Int : Let $x=0$

$y = (0-2)^2 + 3 = 7$

x-Int : None

Domain $= (-\infty, \infty)$

Range $= [3, \infty)$

$f(x) = -\frac{1}{2}(x+2)^2 - 3$

$f(x) = a(x-h)^2 + k$

1) $a = \frac{-1}{2}$ $h = -2$ $k = -3$

2) $a < 0 \Rightarrow$ opens downward

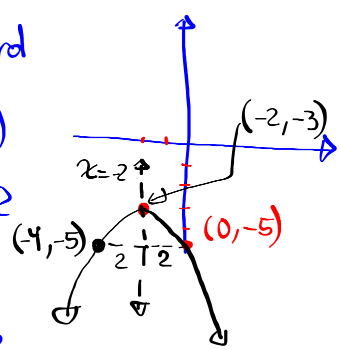
3) Vertex $(h,k) = (-2,-3)$

4) A.O.S. $x = h$ $x = -2$

Y-Int : Let $x=0$, Find y

$y = \frac{-1}{2}(0+2)^2 - 3$

$= -2 - 3 = -5$



x-Int : None

Domain: $(-\infty, \infty)$

Range: $(-\infty, -3]$

$f(x) = 2(x-2)^2 - 8$

$f(x) = a(x-h)^2 + k$

1) $a = 2$ $\quad h = 2$ $\quad k = -8$

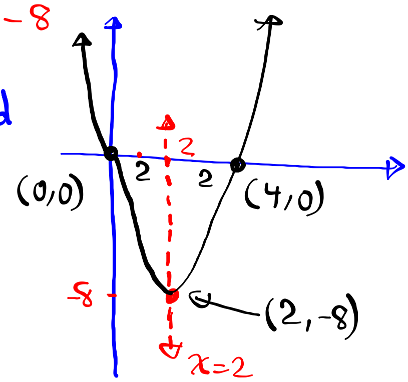
2) $a > 0 \rightarrow$ opens upward

3) Vertex $(h, k) = (2, -8)$

4) A.O.S. $x = h$ $\quad x = 2$

5) Y-Int: $(0, 0)$

$2(0-2)^2 - 8 = 2(-2)^2 - 8 = 8 - 8 = 0$



6) x-Int: $(0,0)$, $(4,0)$

7) Domain $(-\infty, \infty)$ $\quad$ Range $(-8, \infty)$

$f(x) = -(x+3)^2 + 2$

$F(x) = a(x-h)^2 + k$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 2]$

1) $a = -1$ $\quad h = -3$ $\quad k = 2$

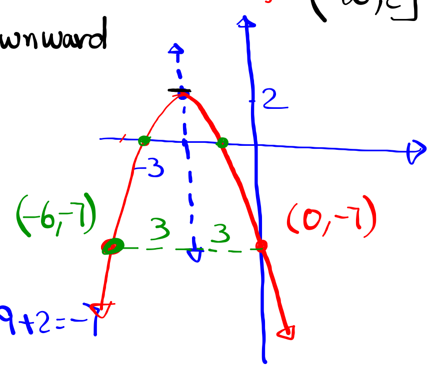
2) $a < 0 \Rightarrow$ opens downward

3) Vertex $(-3, 2)$

4) A.O.S. $x = -3$

5) Y-Int $(0, -7)$

$-(0+3)^2 + 2 = -9 + 2 = -7$



6) x-Int $\quad y = 0 \rightarrow f(x) = 0$

$-(x+3)^2 + 2 = 0$

$-(x+3)^2 = -2$

$(x+3)^2 = 2$

S.R.M.

$x + 3 = \pm\sqrt{2}$

$x = -3 \pm \sqrt{2}$

$(-3+\sqrt{2}, 0)$

$(-3-\sqrt{2}, 0)$

Quadratic Function $f(x)=ax^2+bx+c$; $a\neq 0$

$a>0$ $\cup$ $a<0$ $\cap$

$h=\frac{-b}{2a}$ $K=f(h)$

$x=h$ A.O.S.

$F(x)=x^2-4x$

$a=1$ $b=-4$ $c=0$

opens upward

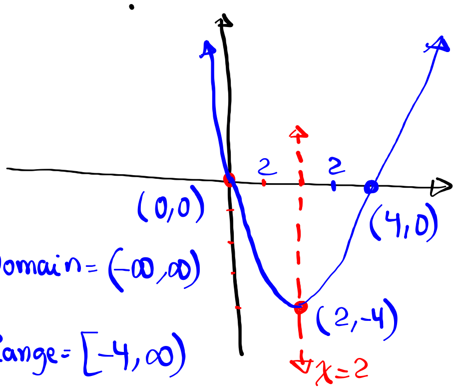
$h=\frac{-b}{2a}=\frac{-(-4)}{2(1)}=\frac{4}{2}=2$

$K=f(2)=2^2-4(2)$
$=4-8=-4$

Vertex $(2,-4)$

A.O.S. $x=2$

Y-Int $(0,0)$



Domain $=(-\infty,\infty)$

Range $=[-4,\infty)$